Journal of Engineering Physics and Thermophysics, Vol. 75, No. 5, 2002

HEAT TRANSFER IN A PLANE THREE-LAYER SYSTEM WITH A HEAT-CONDUCTING NOT-THROUGH ENCLOSURE

A. N. Khutornoi, N. A. Tsvetkov, and S. I. Skachkov

UDC 699.86.022.3+666.198:691.87

Consideration is given to the laws of formation of temperature fields in the structure of the outer three-layer brick wall with the use of efficient warmth-keeping jackets and connectors (flexible couplings). The influence of the connector on the temperature field of the wall has been shown. It has been established that most of the heat arrives at the connector through lateral surfaces before the cross section with a maximum value of the transmission heat beyond the region of possible condensation of a water vapor. After this cross section, the heat is removed from the connector to the materials of the outer wall. Satisfactory agreement between the calculated and experimental results has been obtained.

We analyze heat transfer in a plane multilayer system with a transverse not-through enclosure with the example of a brick three-layer structure of the outer wall with connectors (Fig. 1) with the use of efficient warmth-keeping jackets. For this purpose we arbitrarily subdivide the system in question into ten calculation portions (Fig. 1).

The calculated parameters of the outside t_{out} and inside t_{in} air are constant. The coefficient of heat transfer of the outer and inner surfaces of the structure α_{in} and α_{out} are known (α_{in} = 8.7 W/(m^2·K) and α_{out} = 23 W/(m^2·K)). The thermophysical properties of the brickwork of the inner row λ_{br1} and $(c\rho)_{br1}$, the warmth-keeping jacket λ_j and $(c\rho)_j$, the outer row λ_{br2}, and $(c\rho)_{br2}$, and the connector λ_{core} and $(c\rho)_{core}$ are also known and independent of temperature. The round connector has a radius R and is embedded to a depth δ_{11} in the inner layer of the wall and to a depth δ_{22} in the outer layer. All the geometric parameters of the structure (R, δ_{11}, δ_{22}, δ_{br1}, δ_{br2}, δ_j, L_1, L_2, L_3, L_4, and L_5) are known. Beyond the zone of influence of the connector (for $r \to \infty$), we know the steady-state temperature field of structural layers, which is specified by a broken line (t_{w1}, t_{w2}, t_{w3}, t_{w4}, t_{w4}, and t_{w6}).

The temperature field in each calculation portion is described by the two-dimensional unsteady differential equation of heat conduction in cylindrical coordinates

$$\frac{\partial t_i(x, r, \tau)}{\partial \tau} = a_i \left[\frac{\partial^2 t_i(x, r, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial t_i(x, r, \tau)}{\partial r} + \frac{\partial^2 t_i(x, r, \tau)}{\partial x^2} \right], \tag{1}$$

$$\tau > 0\,;\quad i = \mathrm{I, II, ..., X}\,;\quad 0 \le x_{i=\mathrm{I,VI}} \le L_1\,;\quad L_1 \le x_{i=\mathrm{II,VII}} \le L_2\,;\quad L_2 \le x_{i=\mathrm{III,VIII}} \le L_3\,;$$

$$L_3 \le x_{i=\mathrm{IV,IX}} \le L_4\,;\quad L_4 \le x_{i=\mathrm{V,X}} \le L_5\,;\quad 0 \le r_{i=\mathrm{VI-X}} \le R\,;\quad R \le r_{i=\mathrm{I-V}} \le \infty\,.$$

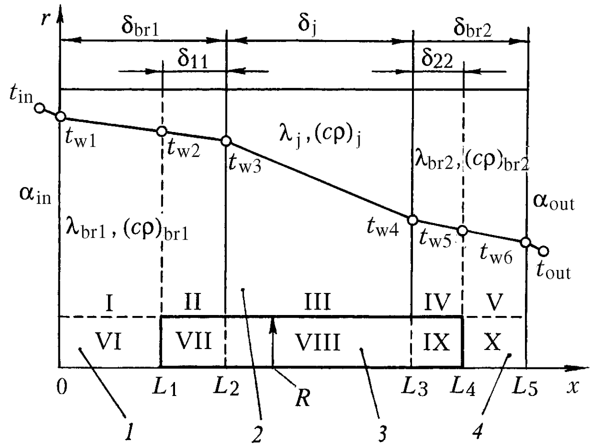
The initial conditions for all the calculation portions are determined by the functions dependent only on the coordinate x:

$$t_i(x, r, \tau)\big|_{\tau=0} = f_i(x)\big|_{r=\infty}\,,\quad i = \mathrm{I - X}\,. \tag{2}$$

It is more convenient to write the boundary conditions in compact form. At the boundary $r = 0$ (for $0 \le x \le L_5$), we have the symmetry condition

State Institution "Scientific-Research Institute of Building Materials" at Tomsk State University of Architecture and Civil Engineering, Tomsk, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 5, pp. 146–148, September–October, 2002. Original article submitted December 11, 2001; revision submitted February 22, 2002.

1062-0125/02/7505-1184$27.00



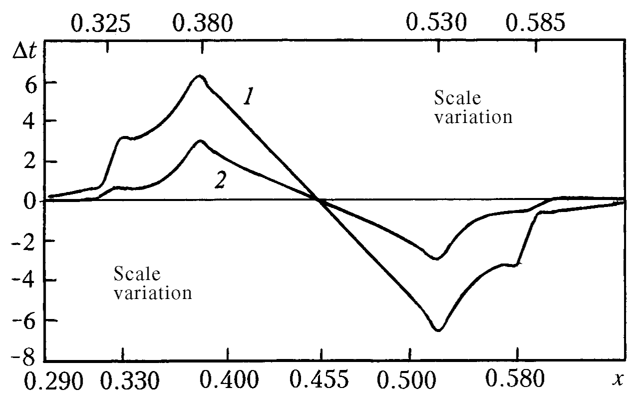


Fig. 1. Problem of heat transfer in the outer multilayer wall with connectors: 1) inner layer of the wall; 2) layer of the warmth-keeping jacket; 3) connector; 4) outer layer of the wall; I–X) calculation portions of the wall.

Fig. 2. Character of distribution of the temperature differences $\Delta t = \{t(x, r)\,|_{r=\infty} - t(x, r)\,|_{r=0}$ for $\lambda_j = 0.05$ W/(m·K), $\delta_j = 0.15$ m, $\lambda_{br1} = \lambda_{br2} = 0.80$ W/(m·K), $\lambda_{core} = 58$ W/(m·K): 1) $R = 4$ and 2) 2 mm. Δt, ^{o}C; x, m.

$$\left.\frac{\partial t_i (x, r, \tau)}{\partial r}\right|_{r=0} = 0\,, \quad i = \mathrm{VI} - \mathrm{X}\,. \tag{3}$$

At the boundary $r = \infty$ (for $0 \le x \le L_5$), the temperature field depends only on the coordinate x:

$$t_i (x, r, \tau)\,|_{r=\infty} = f_i (x)\,|_{r=\infty}\,, \quad i = \mathrm{I} - \mathrm{V}\,. \tag{4}$$

At the boundary $r = R$ (for $0 \le x \le L_5$), the conditions of ideal thermal contact between portions I and VI, II and VII, III and VIII, IV and IX, and V and X are considered to be satisfied:

$$\left.\frac{\partial t_i (x, r, \tau)}{\partial r}\right|_{r=R} = \frac{\lambda_{i+\mathrm{V}}}{\lambda_i} \left.\frac{\partial t_{i+\mathrm{V}} (x, r, \tau)}{\partial r}\right|_{r=R}\,, \tag{5}$$

$$t_i (x, r, \tau)\,|_{r=R} = t_{i+\mathrm{V}} (x, r, \tau)\,|_{r=R}\,, \quad i = \mathrm{I} - \mathrm{V}\,.$$

At the boundary $x = 0$ (for $0 \le r \le \infty$), the heat exchange of portions I and VI obeys the law of convective heat exchange:

$$-\left.\frac{\partial t_i (x, r, \tau)}{\partial x}\right|_{x=0} = \frac{\alpha_{in}}{\lambda_i}\left[t_i (x, r, \tau)\,|_{x=0} - t_{in}\right], \quad i = \mathrm{I}, \mathrm{IV}\,. \tag{6}$$

At the boundary $x = L_5$ (for $0 \le r \le \infty$), the heat exchange of portions V and X also obeys the law of convective heat exchange:

$$-\left.\frac{\partial t_i (x, r, \tau)}{\partial x}\right|_{x=L_5} = \frac{\alpha_{out}}{\lambda_i}\left[t_i (x, r, \tau)\,|_{x=L_5} - t_{out}\right], \quad i = \mathrm{V}, \mathrm{X}\,. \tag{7}$$

At the boundaries $x = L_k$ (k = 1–4), the conditions of ideal thermal contact between portions I and II, II and III, III and IV, IV and V, VI and VII, VII and VIII, VIII and IX, and IX and X are considered to be satisfied:

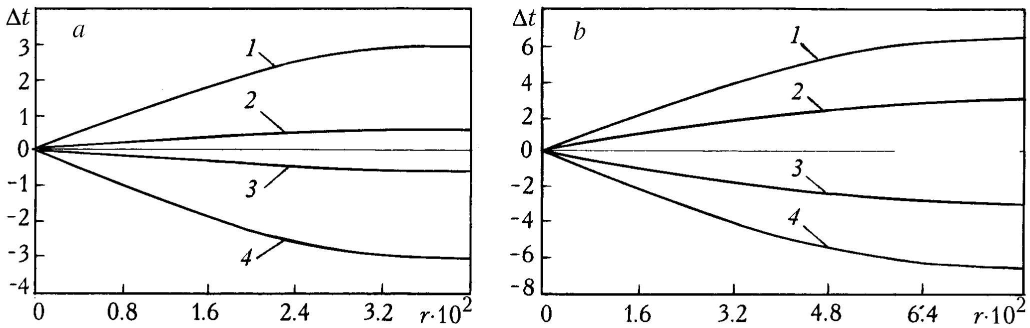


Fig. 3. Character of distribution of the temperature differences $\Delta t = t(x, r) - t(x, r)\big|_{r=0}$ for R = 2 mm (a) and 4 mm (b), λ_j = 0.05 W/(m·K), $\lambda_{br1} = \lambda_{br2}$ = 0.80 W/(m·K), λ_{core} = 58 W/(m·K): 1) x = 0.38, 2) 0.33, 3) 0.58, and 4) 0.53 m. Δt, ^{o}C; r, m.

$$\left.\frac{\partial t_i (x, r, \tau)}{\partial x}\right|_{x=L_k} = \frac{\lambda_{i+\mathrm{I}}}{\lambda_i} \left.\frac{\partial t_{i+\mathrm{I}} (x, r, \tau)}{\partial x}\right|_{x=L_k} , \tag{8}$$

$$t_i (x, r, \tau)\big|_{x=L_k} = t_{i+\mathrm{I}} (x, r, \tau)\big|_{x=L_k} , \quad k = 1 - 4 ; \quad i = \mathrm{I} - \mathrm{IV} , \ \mathrm{VI} - \mathrm{IX} .$$

Numerical solution of the problem formulated has been implemented in the Modula-2 language.

The use of the program developed enabled us to investigate the regularities of the influence of a cylindrical heat-conducting not-through enclosure on the temperature field of a brick three-layer wall.

Of greatest importance are results of calculation of the zones of influence of the connector on the formation of the temperature field of the wall and the direction of the heat fluxes in these zones for different diameters of the steel enclosure. Figures 2 and 3 give such calculation results for the following initial data: $\lambda_{br1} = \lambda_{br2}$ = 0.80 W/(m·K), d_{core} = 0.004 and 0.008 m, λ_j = 0.05 W/(m·K), λ_{core} = 58 W/(m·K), δ_{br1} = 0.38 m, δ_{br2} = 0.12 m, δ_j = 0.15 m, δ_{11} = 0.055 m, δ_{22} = 0.055 m, t_{in} = 20^{o}C, and t_{out} = –40^{o}C.

As is seen in Fig. 2, the largest disturbances of the temperature field are observed in the zones of contact of the inner (x = 0.38 m) and outer (x = 0.53 m) layers of the wall with the warmth-keeping jacket. The connector with a diameter of d_{core} = 0.008 m causes a larger disturbance of the temperature field than the connector with a diameter of d_{core} = 0.004 m. Less substantial disturbances of the temperature field occur near the connector ends.

It has been established that most of the heat arrives at the connector through the lateral surfaces before the cross section with a maximum value of the transmission heat (x = 0.455 m) beyond the region of possible condensation of a water vapor. After this cross section, in the zone of negative temperatures, the heat is removed from the connector to the materials of the outer wall (unlike [2] where it is assumed that heat is supplied to the entire lateral surface of the connector and is removed through the "cold" end).

From Fig. 3a it is clear that in the radial direction of the metallic connector of diameter d_{core} = 0.004 m the zone of influence on the temperature field in the brickwork on the coordinate x = 0.38 and 0.53 m is no longer than 0.04 m, and it is no higher than 0.08 m in radius for the connector of diameter d_{core} = 0.008 m.

The program developed enables us to quite accurately calculate the temperature field of three-layer systems in the zone of influence of not-through cylindrical enclosures.

To evaluate the influence of the connector material on the heat-shielding properties of the wall of the structure in question we have performed an experimental investigation.

In the climatic chamber consisting of the cold zone of volume 30 m^3 and the warm zone of volume 28 m^3, we manufactured and tested a fragment of a three-layer enclosing structure with dimensions 2450 × 2800 × 660 mm.

We embedded 21 glass-reinforced-plastic connectors of diameter d_{core} = 0.006 m and 21 metallic connectors of diameter d_{core} = 0.004 m at 380 mm intervals. The glass-reinforced-plastic connectors were located in the lower part of the fragment in question with an area of 3.43 m^2, while the metallic connectors were located in the upper part on the same area. The thickness of the inner layer of the wall was 380 mm and that of the outer layer was 120 mm; the thickness of the warmth-keeping-jacket layer was 140 mm. A 20-mm plaster layer was applied onto the inner surface of the wall.

The tests were carried out at a temperature of minus 20.1 ± 0.2^oC in the cold zone of the climatic chamber and of plus 20.1 ± 0.2^oC in the warm zone. After the heat transfer reached the steady-state regime, the temperature was plus $18.6–18.9^oC$ at different points on the inner surface of the wall and minus $19.4–19.7^oC$ on the outer surface for six series of tests. The measured densities of the heat fluxes through the tested wall were 11.1 ± 0.3 W/m^2 on the portion with metallic connectors and 10.6 W/m^2 on the portion with glass-reinforced-plastic connectors.

We have calculated the temperature fields for experimental conditions by the numerical method with the use of the program developed and according to the standard procedure [3] and calculated dependences for the steady-state heat transfer [4]. Comparison of the results obtained for the temperature fields and the heat-flux densities yielded their satisfactory agreement (discrepancy is no higher than 6.6%), which enables us to recommend the dependences of [4] for the practice of engineering calculations and makes it possible to use the program developed for a more accurate determination of the temperature fields in the zone of influence of the connectors.

NOTATION

τ, time, k; x, running coordinate over the wall thickness, m; r, running coordinate of the radial direction, m; R, radius of the connector core, m; δ_{br1} and δ_{br2}, thickness of the inner and outer row of the wall, m; δ_j, thickness of the layer of the warmth-keeping jacket, m; δ_{11} and δ_{22}, embedding of the connector in the inner and outer wall layer, m; L_1, L_2, L_3, L_4, and L_5, dimensions of the calculation zones, m; t_{w1}, t_{w2}, t_{w3}, t_{w4}, t_{w5}, and t_{w6}, steady-state temperature field over the wall thickness without allowance for the influence of the connector, ^oC; t_{in} and t_{out}, temperature of the inside and outside air, ^oC; α_{in} and α_{out}, coefficients of heat transfer on the inner and outer wall surfaces, W/(m^2·K); λ_{br1} and λ_{br2}, thermal conductivity of the material of the inner and outer row of the wall, W/(m·K); λ_j and λ_{core}, thermal conductivity of the warmth-keeping jacket and of the connector-core material, W/(m·K); c_{br1} and c_{br2}, heat capacity of the material of the inner and outer row of the wall, J/(kg·K); c_j and c_{core}, heat capacity of the warmth-keeping jacket and of the connector-core material, W/(m·K); ρ_{br1} and ρ_{br2}, density of the material of the inner and outer row of the wall, kg/m^3; ρ_j and ρ_{core}, density of the warmth-keeping jacket and of the connector-core material, kg/m^3; a, thermal diffusivity, m^2/sec; $\alpha = \lambda / c\rho$. Subscripts: br1 and br2, brickwork of the inner and outer row of the wall; j, warmth-keeping jacket; core, connector core; 11 and 22, referring to the inner and outer row of the wall; 1, 2, 3, and 4, Nos. of the calculation zones of the wall; w1, w2, w3, w4, w5, and w6, boundaries of the calculation portions I–V of the wall along the x axis; in and out, inside and outside air; i, No. of the calculation portion of the wall; k, No. of the calculation zone of the wall.

REFERENCES

1. N. A. Tsvetkov, A. V. Grishin, and L. N. Tsvetkova, in: *Ext. Abstr. of Papers presented at Int. Sci.-Pract. Conf. "International Cooperation in the Field of Architecture, Civil Engineering, and Environmental Conservation"* [in Russian], 2–9 November 1997, Kemer, Turkey (1997), p. 61.
2. N. P. Umnyakova, in: *Proc. VII Congr. of the Association of Engineers Dealing with Ventilation, Heating, and Conditioning* [in Russian] (2000), pp. 40–43.
3. *SNiP II-3-79**, *Construction Thermal Engineering*, State Committee of Construction of Russia [in Russian], Moscow (2000).
4. A. N. Khutornoi, N. A. Tsvetkov, and O. I. Nedavnii, *Izv. Vyssh. Uchebn. Zaved., Stroitel'stvo*, No. 6, 13–17 (2000).